Multi-objective Cybernetics and the Concept-based Approach: Will They Ever Meet?

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Abstract. Considering a broad interpretation of cybernetics, we propose to use the term Multi-objective Cybernetics (MOC) when referring to "multi-objective problems in nature and the artificial." In addition to the suggested terminology and its root, the concept-based approach and its motivation is described, as well as an extension of the Pareto-directed concept-based multi-objective problem. The mutual introduction of both MOC and the concept-based approach is not arbitrary since that potential analogies, or at least metaphors, might emerge. Yet, any proposed analogies are likely to be controversial and speculative. In addition to raising the question on the similarities, this paper discusses these issues and suggests future MOC research concerning a possible analogy between concepts and species.

1 Introduction

The main motivation for our work on the concept-based approach is to develop a novel interactive framework to support engineering design. In particular we focus on the lack of tools to link conceptual with preliminary and detailed design stages. In trying to achieve such a goal, and when considering a generalization to problem solving in other application areas, several observations appear significant. First, it should be emphasized that in spite of efforts towards a kind of mechanization of engineering design, it remains one of the most human-related activities featuring intellectualism, creativity and ingenuity [1]. The conceptual design stage should be viewed as a phase with human intensive creativity, whereas the following stages could be viewed as more suitable for mechanization. In general, the cognitive ways, by which humans treat problems by and large involve no formal mathematical definition, but rather conceptualize solutions. In general, a conceptual solution can be viewed as a category. In the concept-based approach it is practically considered as a representative set of the particular solutions, which belong to the category. Humans, and in particular engineers, tend to first select a preferred conceptual solution among several possible ones, and then work out a particular solution out of the selected category. It is also quite often that humans are satisfied with a particular solution which is not necessarily an optimal solution. This is probably not just a result of the frequent lack of models to evaluate the solutions, but it is also related to humans' inability to mentally handle the entire search space. Computers provide a mean to better search the problem space, but they fail to generate conceptual solutions to most non-trivial problems. When trying to develop a synergistic approach between humans and computers, as related to problem solving, and in particular with respect to design, it should also be noted that humans are able to evaluate solutions even without a clear model. This is well reflected in artistic design, and the interactive EC approach [2].

Product development commonly involves trade-offs among contradicting objectives (e.g., accuracy vs. cost). The significance of such trade-offs to creative design has been highlighted in the TRIZ method, which resulted from a comprehensive study of patents by Altshuller [3]. Traditionally multi-objective problems (MOPs) have been treated using either weighted sum of the objectives or a goal attainment approach. Such techniques have a major deficiency involving the need to have a-priori knowledge about the problem, and in particular a-priori narrowing of the objective space by way of some preferences.

Modern processing technologies provide a means to consider parallel search methods which are suitable for range-independent MOP solving (e.g., [4]). In particular EC tools are known to be suitable for supporting engineering design (e.g., [5], [6]). Their attractiveness for engineering design has been strengthened by the recent developments of reliable and generic MOEAs, such as NSGA-II and SPEA2, and by the introduction of interactive EC methods for engineering design such as COGA (See recent reviews in [7] and [8] by Coello, and Parmee respectively). Pareto-based search has been implemented for engineering design and other applications by non EC methods (e.g., [9]). Yet, it appears that in recent years Evolutionary Multi-Objective Optimization (EMOO) techniques are becoming the most popular methods to solve MOPs. It should be noted that the majority of EMOO studies employs a traditional MOP approach rather than a Concept-based MOP (C-MOP) approach, as discussed in [10], and described here.

The underline idea of the concept-based approach is that the search and optimization is carried out by way of feasible sub-sets (concepts) of their associated decision spaces (e.g., [9-12]). This is in contrast to traditional MOPs that are commonly defined without such a distinction. The proposed distinction may look somewhat arbitrary as one may claim that a-priori division of a decision space to decision spaces and associated sub-sets is immaterial to the search and optimization problem as all possible solutions should be equally compared. Yet, a clear statement of C-MOPs is essential. This is due to the concept search related issues, such as resource sharing among concepts and preservation of representatives of superior concepts throughout the search [10]. We view C-MOPs to be generic problems as demonstrated by an implementation to a path planning problem [13]. One may define a C-MOP in deferent ways, which reflect different approaches to MOPs, including both range-dependent and range-independent ones (e.g., [11], [13] respectively). To demonstrate the notion of a C-MOP, a new definition, which is an extended version of the one in [9], is given in the appendix. The proposed extension might play a role in our future studies as related to the raised intriguing discussion on the metaphors and analogies between evolutionary theories and MOP-related EC. Here we highlight such issues and provide suggestions on Multi-objective Cybernetics (MOC) research needs concerning a possible analogy between concepts and species. The paper is organized as follows. It starts with a description on the concept-based approach and the extended C-MOP. Then the paper introduces the proposed MOC term and its root. Finally, the concept-based approach is discussed in view of MOC and the raised question of the paper title.

2 The Concept-based Approach

Traditionally search and optimization techniques have been used with conceptual solutions that have each a one-to-one relationship with a point in the objective space. This is not a surprise when considering human tendency to (at best) provide a concept with just a one-to-one relationship with the objective space, reflecting their experience and understanding of the concept and its overall performance. This approach is inherent to most methods that deal with the selection of optimal concepts (for a review see [9]). The concept-based approach deviates from this tradition as explained below. Any C-MOP involves the notion of conceptual solution, which is represented by a set of particular solutions. In contrast to the traditional approach, a C-MOP involves a one-to-many relationship between a conceptual solution and the objective space. This means that a C-MOP allows performance variability among the particular solutions, which are associated with a conceptual solution. The first Pareto-based C-MOP definition has only recently been formalized by Mattson and Messac in [9], and is extended in the current appendix. They have chosen the term s-Pareto to designate that the problem is set-based. Here, s-Pareto is renamed as C-Pareto to clarify that the set represents conceptual solutions. Variants of the C-MOP have been treated elsewhere without a formal definition (e.g., [12], [13]). The contribution of the concept-based approach, which has been developed at TAU, covers several elements including:

- A concept-based dynamic goal approach (e.g., [11])
- Novel concept-based MOEAs which are inspired by NSGA-II (e.g., [10])
- A study on sequential vs. simultaneous concept-based search [10]
- A structured evolutionary approach using sub-concepts (e.g., [14])
- A study on concept-based simultaneous mechanics and control design [11]
- Interactivity by preferences of concepts and sub-concept & subjective-objective fronts (e.g., [14])
- Generalization by demonstration to concept-based path planning ([13])
- Non-traditional types of robust concepts and related algorithms (e.g., [15])

Much of the referred studies and our current efforts concern measures of optimality and robustness of concepts in the objective space, and the related algorithms. We have demonstrated the evaluation of concepts both by the use of only computable models (in [10]), and by the use of a mixture of computable models with mental models (e.g., [11]). It is assumed that the concept-based approach can also be attempted with experimental-based evaluations as done in evolutionary robotics.

The general motivation and logic behind the concept-based approach is given in the introduction. One may still wonder why defining C-MOPs (as demonstrated in the appendix), and providing new MOEAs for their solution is significant at all. The wondering opponent may insist that designers are searching for a (particular) solution and not a set of solutions (concept). Furthermore, if particular solutions are available to represent concepts then why should one 'look back' to concepts? Similarly one may (re)ask why is it important to find the Pareto-set when eventually only one solution is expected to be implemented. The case of the Concorde aircraft design provides a good example for the significance of the proper choice of a concept. The Concord design constitutes an excellent product with superb performance but as a concept it could be considered a failure! In modern engineering the proper evaluation and selection of concepts might make the difference between a flourishing company and a disappearing one. Humans have the ability to evaluate concepts, regardless of the lack of a clear model with a risk of making a substantial mistake (see a review on methods in [9]). Most of concept selection methods assume some vague preference values to be decided by the designers based on their experience. The concept-based approach involves a novel representation and evaluation of concepts by way of their multiple representatives in the objective space. This allows concepts to be evaluated by concrete computable models that are available in many engineering problems. Such a computer-supported evaluation could be related to both optimality and robustness of concepts. Furthermore, the concept-based approach provides understanding of the relations not only between concepts but also with respect to their particular solutions. The need for a concept-based search and optimization becomes apparent when considering modern concurrent design approaches such as the concept of families of design and the concept of delaying decisions. The latter concept is perhaps not an obvious advantage, and the reader is therefore referred to a comprehensive discussion in [17] on delaying decisions during product development. Designers should pay careful considerations not only to the selected detailed design but also to its selected concept, (and its influence on the company survivability). This justifies the development of the concept-based approach and the search of both optimal and robust concepts.

It should be noted that in engineering design the selected solution might not necessarily be from the Pareto-optimal set as emphasized by the COGA approach and in the interactive concept-based approach ([8], [14]). Yet, an understanding of the concepts' relative performances along such a front is significant to solution selection. This is illustrated in figure 1. Assume that the figure contains the performances of all solutions of two concepts. Both concepts (designated by stars and circles) play a role in the front. Yet, when a look beyond the front is taken, the "star concept" might be more robust, as related to the upper part of the front, when the solutions of the first two ranks are to be disregarded due to some uncertainties. The last observation serves as a motivation for the extended version of a Pareto-based C-MOP (eC-MOP), which is introduced here.

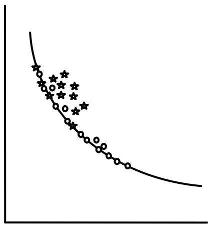

Fig. 1: Concepts performances in the objective space

The eC-MOP is based on a combination of two ideas, namely the definition of the Pareto-based C-MOP, in [9], and the notion of ε-dominancy, (e.g., [16]). The exposition of the eC-MOP, as outlined in the appendix, provides a platform, which seems more resilient than the C-MOP for concept comparisons. Finally, in view of section 4, on comparing concepts and species, it also appears significant to relax the optimality condition.

3 Multi-objective Cybernetics

The traditional definition of cybernetics, as the science of communication and control in the animal and the machine is attributed to Norbert Weiner [18]. The fathers of cybernetics, such as Weiner, studied analogies and metaphors between animals and machines starting at the level of a neuron up to and including the level of societies. It should be pointed out that there are a host of different definitions to cybernetics as listed by the American Society for Cybernetics (www.asc-cybernetics.org/foundations/definitions.htm). Of special interest here are the non-traditional ones such as: "the art and science of manipulating defensible metaphors" (by Gordon Pask), "the art of securing efficient operation" (by Luis Couffignal), "...[the] mathematical and constructive treatment of general structural relations, functions and systems" (by F. von Cube), and "the art and science of human understanding" (by Humberto Maturana). It is clear from this collection that cybernetics can be viewed from different and much broader perspectives than that of the original one. Following Couffignal, we view modern cybernetics as *the study of assuring efficiency of action with a focus on goals and within the scope of analogies of modern cybernetics* as schematically depicted in Figure 2.

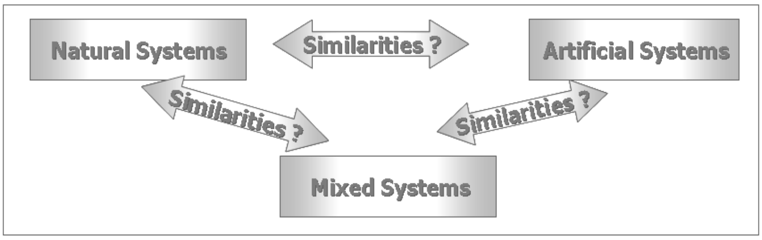

Fig. 2: The scope of modern cybernetics

We also hold the modern view that understanding “communication and control” should not be separated from understanding “morphology and mechanics.” In fact, evolution appears to suggest a mixed view on the 'How' and 'What' is 'Governed' and 'Governing.' Cybernetics includes two interesting view-points when referring to figure 2. The first view of cybernetics is aimed at studying natural systems to support the development of better man-made systems (with arrows to the right), whereas the second view-point involves using new ideas, which are generated as a part of the development of artificial systems, to possibly find new explanations to nature (left pointing arrows). As examples of the former consider EC and modern robotics, and for the latter consider the use of EC in explaining natural evolution as done in [19].

Assuring efficiency of action with a focus on goals, as given in our definition above, suggests an extension to multi-objectiveness. The term Multi-objective Cybernetics has been originated by the author as a result of working on the concept-based approach, and his realization of the metaphorical interpretation of concepts as species. An intriguing question had to be raised, namely, given that the notion of 'survival of the fittest' is possibly related to single objective optimization (in both cybernetic view points) then could this relation be extended when multi-objective optimization is concerned. It should, however, be noted that putting adaptation and optimization together is a subject of a controversy primarily with respect to the 2nd MOC point of view [20]. Several related observations are made in the following:

- Pareto ideas were not available at the time of Darwin's 'origin of species.' Yet the following is quite surprising.
- There is no reference on a multi-objective evolutionary theory
- The notion of objectives is controversial with respect to nature, and the equivalent notion might be that of trade-offs of functions and forms. Yet, there is no well-known theory (to the author) of evolution that relates fitness with trade-offs of functions and forms (or alike).
- There is, however, evidence for trade-offs in biological systems (e.g., [21]).
- A related attempt is that of adopting the TRIZ methodology (see [3]) to a 'biological patents' database, which is carried out at the University of Bath [24]. It might shade light on possible analogies as related to trade-offs.
- There is also an emerging evidence on studies of trade-offs that is found in the field of Evolutionary Robotics (e.g., [23])
- There is an increasing evidence of the use of multi-objective optimization in bioinformatics and computational biology (see a recent review in [22]), yet much of this could be viewed as engineering-related activities.

The above list calls for a comprehensive discussion on the validity of MOC analogies, and possible metaphors. The most fundamental aspect of the common attempts to compare evolution theories and EC is the notion of "the fittest" that is easily associated with the idea of "optimal solution." In fact the 'analogy' does not stop there. Terms such as survival of the fittest, population, crossover, mutations, niching, speciation, co-evolution, islands, and migration have been most useful in describing the rational of EC. Yet, most of the explanations that exist in the common EC literature do not specifically refer to MOPs. For example, what does "survival of the fittest" means when a MOP is concerned? In MOEA literature there has been some explanation on modifying the fitness (e.g., as referred to NSGA and SPEA).

Yet, there is no clear linkage to the meaning of such a modification with respect to nature, and to the validity of any possible analogy that concerns the modified fitness and the known evolutionary theories. We don’t aim at adding to the 'adaptation and optimization' controversy (see [20]), but rather to provide a framework of thinking when comparing natural and artificial systems. We view MOC as such a framework when considering "multi-objective problems in nature and the artificial." One may view MOC as *the science that focuses on multi-'objective' evaluation and selection with respect to assuring efficiency of action.* The notion of an action should be viewed as a generic one (e.g., an action towards solving a MOP is included). The proposed MOC definition requires a through discussion to relate it to the various definitions of cybernetics, and to provide it with a clear justification, which is beyond the scope of this paper.

4 Comparing Concepts and Species

Understanding analogies and metaphors between the natural and the artificial, as related to MOPs, seems important from a broad cybernetic sense, yet such an attempt is inherently difficult and mostly speculative at this stage. The prime merit of the following is perhaps in raising some questions and pointing at potential approaches that have resulted from our research on the concept-based approach in engineering design. Speculation could be avoided by focusing on possible analogies as a means for possible inspiration and metaphors, without trying to pose any new theories on nature. Yet, at the risk of being mocked at a two way MOC view is attempted in the following, regardless of the well known controversy on 'adaptationism and optimality' [20].

In fact, understanding that an analogy between design concepts and species might exist had an important impact on the development of our algorithms in [10]. The following provides some background to understand the potential relations. In biology the term species commonly refers to the most basic biological classification comprising of individuals that are able to breed with each other but not with others (except from rare cases). In nature, a niche can be viewed as a subspace in the environment with finite resources that must be shared among the population (society) of that niche, while competing to survive. In evolutionary algorithms the term speciation (or "niching") commonly refers to an automatic technique to overcome the tendency of the population to cluster around one optimal solution in a multi-modal function optimization. Speciation techniques help maintaining diversity to prevent premature convergence, while dealing with multi-modality. Speciation could be viewed as an automatic process, or an operator, that gradually divides the population into sub-populations (species). Each of these sub-populations deals with a separate part of the problem ("niche" of the search space). Commonly 'niche' refers to an optimum of the domain and the fitness represents the resources of that niche. The common process of speciation is also a niching process as it finds the niches, while dividing the population into the niches.

Species that are either competing or cooperating are viewed as co-evolving. Competitive co-evolution has been computationally employed with single as well as with multi-populations. In contrast to niching, where species are automatically formatted, in co-evolution of competing species, the species are commonly predefined (although their populations' relative size may be subject to automatic changes). This situation resembles that of C-MOPs, in which the association of sets of particular solutions with concepts is predefined. The last observation clearly indicates a possible analogy between concepts and species. Both are represented by sub-sets of the populations. Beyond the mathematical similarities, it seems natural to view different species as different "design concepts of nature."

A crucial part of the algorithm in [14], which simultaneously evolves concepts towards and along the C-Pareto front, is the penalty functions that are used for the fitness. These include a front-based concept-sharing penalty and an in-concept front niching penalty. The front-based concept sharing is applied to preserve concept diversity, and to prevent a good concept from hindering the evolution of other potential concepts within a front. The in-concept front niching preserves the diversity of particular solutions within each concept belonging to a particular non-dominated front (rank). In a recent investigation the algorithm of [14] has been modified to improve the analogy by eliminating crossover operations between concepts [10]. In the latter publication a crowding approach has been implemented to penalize the fitness. In developing these penalties, and the entire algorithm, the focus has been on engineering design and the wish to find a good representation of the optimal concepts. With the elimination of crossover operations between concepts, it appears that the process of the simultaneous concept-based EMOO could be viewed as the evolution of species towards and along a Pareto front. Efforts such as described here might open up a unique avenue for exploring scenarios of evolution and their simulations in the sense of both the 1st and the 2nd view point of MOC.

While supporting the development of computational mechanisms to simultaneously evolve species/concepts towards and along a Pareto front, by the artificial selection pressures of the EC approach, a host of questions should be raised as to the applicability of such comparisons with respect to improving the understanding of nature. The main question from the 2nd view point of MOC is to what a degree it would be possible to advance the potential analogy between design concepts and species to obtain better understanding of evolution at-large (under the controversy constraints as discussed in [20], and the promises of the TRIZ approach [24], and related studies on evolution such as in [21]). Furthermore, it is still questionable if new metaphors might arise from taking a MOC view rather than a single objective view on nature. Clearly, the algorithms in our studies have been developed for engineering design application and not as simulators of natural selection. Yet, as listed in the section on MOC, multi-'objective' situations (in the sense of multi-performance or multi-functionality) do exist in nature. Very basic survival situations in nature could involve trade-offs in behaviors such as fast (to obtain food) versus safe (to avoid dangers), which has been the subject of our robotic-related study in [13]. Different evolutionary situations could exist over different times (and time scales) and locations, which would require a different solution along and towards a Pareto-front. Incorporating such evolution scenarios into the concept-based approach might create a new way of studying natural evolution in the sense of the 2nd view point of MOC. The following is an open question for future research. Would it be possible to say that, regardless of different scenarios, nature evolve species towards optimality in a multi-objective sense, just as humans trying to create conceptual designs that are satisfying in some Pareto sense?

As hinted at the two first sections, engineering design often involves satisfying solutions that are not necessarily Pareto-optimal. Similarly, it is expected that natural selection involves 'design solutions' that could be viewed as advancing towards a Pareto-front, but are not optimal in the Pareto sense. With this respect it appears logical, from both view points of MOC, to develop modifications of the C-MOP as suggested here. Currently we are also investigating a novel 'exile' metaphor, which has been suggested by the author to help improve the algorithms of the concept-based approach with respect to local Pareto issues. Such efforts raise the question on the applicability of the notion of 'non-dominated' sets in natural evolution, and its role in natural diversity.

All of our ongoing research efforts and agenda on the concept-based approach have been strongly influenced by our attempts to better understand possible analogies between nature and the artificial. Of a particular interest for future research is to investigate potential analogies and metaphors with respect to our studies on the robustness of concepts (e.g., [15]). This topic encompasses different types of robustness with respect to different types of uncertainties, and requires the introduction of measures not only for multi-objective optimal concepts, but also for their robustness. With this respect, methods of comparisons, in the multi-objective sense, of particular solutions and of concepts (sets), as well as their rational, might also serve as a MOC research playground when such questions are asked with respect to species. A more questionable idea is to try and compare our studies on the interactivity aspects of the concept-based approach with evolutionary issues of mixed systems (see figure 2). Finally, it should be noted that due to the fact that the concept-based approach is a set-based approach analogies might be explored not only with respect to species but also with respect to other biological categories.

5 Summary and Conclusions

This paper deals with a concept-based approach to MOPs and its potential relations with the evolution of species. As a part of the discussion a new term is suggested, namely MOC, and an extension of the Pareto-based C-MOP definition is provided. Several questions are raised, which are related to a long standing controversy on adaptationism and optimality. At the risk of falling into this controversy the paper suggests a comparison between species and engineering design concepts and hints at possible analogies with respect to their multi-objectiveness. In addition, future MOC research directions are proposed. It is concluded that MOC is a justified framework of thinking that has a ground in past and present findings both in engineering design research and biology. Yet, its scope, as demonstrated here, is bound to be controversial, which makes it both an intriguing and exciting research area.

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References

1. Horvath, I.: A Contemporary Survey of Scientific Research into Engineering Design. Proceedings of the International Conference on Engineering Design, ICED 01 Glasgow, (2001).
2. Takagi, H.: Interactive Evolutionary Computation: Fusion of the Capacities of EC Optimization and Human Evaluation. Proc. of the IEEE, 89(9), (2001) 1275-1296.
3. Savransky, S.D.: Engineering of Creativity: Introduction to TRIZ Methodology of Inventive Problem Solving. CRC Press (2000).
4. Srinivas, N., Deb, K.: Multi-objective Function Optimization Using non-dominated Sorting Genetic Algorithms. Evolutionary Computation, 2, 3 (1995) 221-248.
5. Bentley, P.J. (Ed.): Evolutionary Design by Computers. Morgan Kaufmann, San Francisco, California, (1999).
6. Kicinger R., Arciszewski T., De Jong K.: Evolutionary Computation and Structural Design: a Survey of the State of the Art. Computers and Structures, 83; 23-42; (2005) 1943-1978.
7. Coello, C.A.C.: Recent Trends in Evolutionary Multiobjective Optimization. in A., Abraham, L. Jain and R. Goldberg (eds.), Evolutionary Multiobjective Optimization: Theoretical Advances And Applications, Springer-Verlag, London, (2005) 7-32.
8. Parmee, I.C.: Human Centric Intelligent Systems for Design Exploration and Knowledge Discovery. Proc. of ASCE 2005 Int. Conf. on Computing in Civil Eng. Cancun, Mexico (2005).
9. Mattson, C. A.: Messac, A.: Pareto Frontier Based Concept Selection under Uncertainty with Visualization. Optimization and Engineering, 6; (2005) 85–115.
10. Avigad, G., Moshaiov, A.: Simultaneous Concept-based EMO. A technical report, available from www.eng.tau.ac.il/~moshaiov, also submitted to IEEE Trans on EC (2006).
11. Avigad, G., Moshaiov, A., Brauner, N.: Concept-based Interactive Brainstorming in Engineering Design. Journal of Advanced Computational Intelligence and Intelligent Informatics, 8; 5; (2004) 454-459.
12. Anderson, J.: Multiobjective Optimisation in Engineering Design Applications to Fluid Power Systems. Ph.D. Thesis, Linkoping University, Sweden, (2001).
13. Moshaiov, A., and Avigad, G.: Multi-objective Path Planning by the Concept-based IEC Method. Proceedings of the 2004 IEEE Int. Conference on Computational Cybernetics, ICCC 2004. Vienna, Austria, (2004).
14. Avigad G., Moshaiov A., Brauner N.: Interactive Concept-based Search using MOEA: The Hierarchical Preferences Case. Int. J. of Computational Intelligence, 2; 3; (2005) 182-191.
15. Moshaiov A., Avigad, G.: Concept-based IEC for Multi-objective Search with Robustness to Human Preference Uncertainty. Proceedings of the IEEE Congress on Evolutionary Computations, Calgary, Canada, (2006).

16. Laumanns, M., Thiele, L., Deb, K., and Zitzler, E.: On the Convergence and Diversity-Preservation Properties of Multi-Objective Evolutionary Algorithms. TIK-Report No. 108, Swiss Federal Institute of Technology (ETH) Zurich, Switzerland (2001).
17. Sobek, D.K., Ward, A.C.: Principles from TOYOTA'S Set-based Concurrent Engineering Process. Proceedings of the 1996 ASME Design Engineering Technical Conferences and Computers in Engineering Conference, August 18-22, Irvine, California (1996).
18. Weiner, N.: Cybernetics or Control and Communication in The Animal and The Machine. MIT Press (1948).
19. Dawkins, R.: The Blind Watchmaker. Longman Scientific and Technical, Harlow, (1986).
20. Abrams, P.: Adaptationism, Optimality Models, and Tests of Adaptive Scenarios, in Adaptationism and Optimality. S.H. Orzack, and E. Sober (eds.), Cambridge University Press (2001).
21. Farnsworth, K.D., Niklas, K.J.: Theories of Optimization, Form and Function in Branching Arhitecture in Plants. Functional Ecology, 9, (1995) 355-363.
22. Handl, J., Kell, D.B., Knowles, J.: Multiobjective Optimization in Bioinformatics and Computational Biology. Technical Report, TR-COMPSYSBIO-2006-04, available at www.dbkgroup.org/handl/, (2006).
23. Teo, J., Abbass, H. A.: Multiobjectivity and Complexity in Embodied Cognition. IEEE Trans. on Evolutionary Computation, 9, 2, (2005) 337-360.
24. Bogatyreva, O., Pahl, A-K, Vincent J.F.V.: Enriching TRIZ with Biology, The Biological Effects Database and Implications for Teleology and Epistemology. ETRIA World Conf., Strasbourg, (2002) 301-307.

Appendix: Formulizing the Extended Concept-based MOP

The focus here is on obtaining the set of particular solutions of the concepts that have performances along the Pareto-front and its vicinity. Each conceptual solution, and its associated particular solutions, may be characterized by different models and/or variables. Yet, they are examined with respect to the same objectives. We define n_c sets of decision variables; one set for each concept. The m-th set of the feasible solutions of the m-th concept, is denoted X_m, where $X_m \subseteq S_m \subseteq R^{n_m}$, and S_m is the feasible search space of the m-th concept. X_m contains the feasible decision variable vectors, x^m, of the m-th concept, as follows

$$\left\{ x^m \in X_m \middle| \; X_m \subseteq S_m \subseteq R^{n_m} \right\} \quad m = 1,\ldots,n_c \qquad (1)$$

where the dimension n_m of the vectors, x^m, is in general concept dependent. The set X is the union of these n_c sets such that

$$\left\{ x \in X \;\middle|\; X = \bigcup_{m=1}^{n_c} X_m \right\} \qquad (2)$$

The vector of objective functions $F: \; X \rightarrow Y$ is given as follows

$$F(x) = \left\{ F^m\left(x \right) , \quad for \;\; x = x^m, \;\; m = 1,\ldots,n_c \right\} \qquad (3)$$

where $F^m(x^m) = [f_1^m(x^m), f_2^m(x^m), ..., f_K^m(x^m)]^T: X_m \to Y$, for m = 1,..., n$_c$, is the mapping, into the objective space, of the particular solutions that are associated with the m-th concept and

$$\left\{ y \in Y \;\middle|\; y \in R^K, k \geq 2 \right\}. \qquad (4)$$

The mapping of the m-th concept is done using a set of concept related objective functions with $f_k^m(x^m)$ as the k-th objective function. Under minimization a vector $u = (u_1, ..., u_k)$ is said to dominate $v = (v_1, ..., v_k)$, denoted by $u \preceq v$, iff *u* is partially less than *v*, i.e., $\forall i \in \{1, ..., k\}, u_i \leq v_i \wedge \exists i \in \{1, ..., k\} : u_i < v_i$.

Similarly, a vector $u = (u_1, ..., u_k)$ is said to ε-dominate v, denoted by $u \prec_\varepsilon v$, iff $\forall i \in \{1, ..., k\},\ u_i - \varepsilon_i \leq v_i, \quad and\ \varepsilon_i > 0$. Without losing generality the eC-MOP is defined with respect to the min-min optimization problem as:

$$\min_\varepsilon F(x), \text{ s. t. } \quad x \in X \qquad (5)$$

The minimization operator uses the notion of the ε-dominancy to find the eC-Pareto set, P^*_{eC} , and its associated extended "front" PF^*_{eC} , which are defined via the C-Pareto set, P_C^+ and its front, PF_C^+ , as follows.

$$\begin{gathered} P_C^+ := \{x_m^+ \in P_C^+ \subseteq X \mid \neg\exists\, x_i' \in X_i : F^i(x_i') \preceq F^m(x_m^+),\ m \in \{1, ..., n_c\}\ and\ i = 1, ..., n_c\} \\ PF_C^+ := \{y^+ \in PF_C^+ \mid y^+ = F^m(x_m^+) \quad : x_m^+ \in P_C^+\} \end{gathered} \qquad (6)$$

$$\begin{gathered} P_{eC}^* := \{x_m^* \in P_{eC}^* \subseteq X \mid \forall\, x_m^*\ \exists\, x_i^+ \in P_C^+ : F^m(x_m^*) \prec_\varepsilon F^i(x_i^+),\ m \in \{1, .., n_c\}\ and\, i = 1, .., n_c\} \\ PF_{eC}^* := \{y^* \in PF_{eC}^* \mid y^* = F^m(x_m^*) \quad : x_m^* \in P_{eC}^*\} \end{gathered} \qquad (7)$$

The eC-MOP, which is defined above, allows humans to define their vicinity of interest with respect to the behavior of concepts near the Pareto front. This problem definition, as explained in the body of the paper, is natural to the exploration of concepts.